Co - Funding and Joint International Emissions Trading

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Abstract: The conferences of Rio de Janeiro 1992 and Kyoto 1997 demand for new economic instruments which have a focus on environmental protection in the macro and micro economy. An important economic tool being part of the treaty of Kyoto in that area is *Joint-Implementation*. It is an international program which intends to strenghten international cooperations between enterprises in order to reduce CO_2-emissions. A sustainable development can only be guaranteed if the instrument is embedded into an optimal energy management. For that reason, the Technology-Emissions-Means (TEM) model was developed, giving the possibility to simulate such an extraordinary market situation. In this paper, the Kyoto Game is introduced and a first approach to Joint International Emissions Trading (JET) is given.

1 Introduction

The realization of Joint-Implementation (JI) is determined by technical and financial constraints. In a JI Program, the reduced emissions resulting from technical cooperations are recorded at the *Clearing House*. The TEM model integrates both the simulation of the technical and financial parameters. In Pickl (1999) the TEM model is treated as a time-discrete control problem. Furthermore, the analysis of the feasible set is examined in Pickl (2000). In the following, a short introduction into the TEM model is given. Furthermore, we want to present a new bargaining approach which leads towards a procedure for an international emissions trading procedure within the so-called Kyoto game.

2 The Technology-Emissions-Means (TEM) model

The presented TEM model describes the economic interaction between several actors (players) which intend to maximize their reduction of emissions (E_i) caused by technologies (T_i), by expenditures of money (M_i) or by financial means. The index stands for the i-th player, $i \in \{1, \dots, n\}$. The players are linked by technical cooperations and the market.
The effectivity measure parameter em_{ij} describes the effect on the emissions of the i-th player if the j-th actor invests money for his technologies. We can say that it expresses how effective technology cooperations are (like an innovation factor), which is the central element of a JI Program. The variable φ can be regarded as a memory parameter of the financial investigations, whereas the value λ_i acts as a growth parameter. For a deeper insight see Pickl (1999). The TEM model is represented by the following two equations:

$$E_i(t+1) = E_i(t) + \sum_{j=1}^{n} em_{ij}(t) M_j(t) \qquad (1)$$

$$M_i(t+1) = M_i(t) - \lambda_i M_i(t)[M_i^* - M_i(t)]\{E_i(t) + \varphi_i \Delta E_i(t)\} \qquad (2)$$

It is a great advantage of the TEM model, that we are able to determine the em_{ij}-parameter empirically. In the first equation, the level of the reduced emissions at the $t+1$-th time-step depends on the last value plus a *market effect*. This effect is represented by the additive terms which might be negative or positive.

In general, $E_i > 0$ implies that the actors have yet reached the demanded value $E_i = 0$ (normalized *Kyoto-level*). A value $E_i < 0$ expresses that the emissions are less than the requirements of the treaty. In the second equation we see that for such a situation the financial means will increase, whereas $E_i > 0$ leads to a reduction of $M_i(t+1)$:

$$M_i(t+1) = M_i(t) - \lambda_i M_i(t)[M_i^* - M_i(t)]\{E_i(t) + \varphi_i \Delta E_i(t)\}$$

The second equation contains the logistic functional dependence and the memory parameter φ_i which describes the effect of the preceeding investment of financial means. The dynamics does not guarantee, that the parameter $M_i(t)$ lies in the interval, which can be regarded as a budget for the i-th actor. For that reason we have to add the following restrictions to the dynamical representation:

$$0 \leq M_i(t) \leq M_i^*, \quad i = 1, \ldots, n \quad \text{and} \quad t = 0, \ldots, N.$$

These restrictions ensure that the financial investigations can neither be negative nor exceed the budget of each actor. Now, it is easy to show that

$$-\lambda_i M_i(t)[M_i^* - M_i(t)] \leq 0 \quad \text{for} \quad i = 1, \ldots, n \quad \text{and} \quad t = 0, \ldots, N.$$

We have guaranteed that $M_i(t+1)$ increases if $E_i(t) + \varphi_i \Delta E_i(t) \leq 0$ and it decreases if $E_i(t) + \varphi_i \Delta E_i(t) \geq 0$. Applying the memory parameter φ_i, we have developed a reasonable model for the *money expenditure - emission* - interaction, where the influence of the technologies is integrated in the *em*-matrix of the system.

We can use the TEM model as a time-discrete model where we start with a special parameter set and observe the resulting trajectories. Normally, the actors start with a negative value, i.e., they lie under the baseline mentioned in Kyoto Protocol, see Kyoto (1997). They try to reach a positive value of E_i. By adding control parameters, we enforce this development by an additive financial term. For that reason the control parameters are added only to the second equation of our model:

$$M_i(t+1) = M_i(t) - \lambda_i M_i(t)[M_i^* - M_i(t)]\{E_i(t) + \varphi_i \Delta E_i(t)\} + u_i(t)$$

The introduction of the control parameter $u_i(t)$ implies that each actor makes an additional investigation at each time-step. In the sense of environmental protection, the aim is to reach a state, mentioned in the treaty of *Kyoto*, by choosing the control parameters such that the emissions of each player become minimized. The focus is the realization of the necessary optimal control parameters via a played cost game, which is determined by the way of actors cooperation.

3 The Cost-Game in the TEM Model

Let us regard the nonlinear time-discrete dynamics of the TEM-model

$$\begin{aligned}
E_i(t+1) &= E_i(t) + \sum_{j=1}^{n} em_{ij}(t) M_j(t) && (1)\\
M_i(t+1) &= M_i(t) - \lambda_i M_i(t)[M_i^* - M_i(t)]\{E_i(t) + \varphi_i \Delta E_i(t)\} &&\\
&\quad \text{we can also formulate} &&\\
E_i(t+1) &= E_i(t) + \sum_{j=1}^{n} em_{ij}(t) M_j(t) && (2)\\
M_i(t+1) &= M_i(t) - \lambda_i M_i(t)[M_i^* - M_i(t)]\{E_i(t) + \varphi_i \sum_{j=1}^{n} em_{ij}(t) M_j(t)\} &&\\
&\quad \text{considering that} \quad \Delta E_i(t) = E_i(t+1) - E_i(t) &&
\end{aligned}$$

In order to reach steady states, which are determined in Pickl (1999), an independent institution may influence the *trade relations* between the actors. The trade relations are expressed by the *em*-matrix. In practice, the imposing of *taxes* or the giving of *incentives* means that in the TEM-model the em-parameter will change.
Now, the principle of JI implies that technical cooperation will be benefitted. If there is a cooperation between player 1 and player 2, we introduce an additional parameter $\epsilon, \epsilon > 0$, which implies that the measure of effectivity increases. The cooperation of the great coalition is expressed by the parameter ω:

$$\begin{pmatrix} em_{11} & em_{12}+\epsilon & em_{13} \\ em_{21}+\epsilon & em_{22} & em_{23} \\ em_{31} & em_{32} & em_{33} \end{pmatrix}$$

Actor 1 and Actor 2 do cooperate

$$\begin{pmatrix} em_{11}+\omega & em_{12}+\omega & em_{13}+\omega \\ em_{21}+\omega & em_{22}+\omega & em_{23}+\omega \\ em_{31}+\omega & em_{32}+\omega & em_{33}+\omega \end{pmatrix}$$

All players do cooperate

This extension of the TEM model results in a cost-saving effect at each time-step, which can be expressed by an cooperative cost-game. According to (1) and (2) let us begin with the construction of the cost-game in the TEM-model

$$\begin{aligned} v_t(K) &:= \underbrace{\sum_{j\in K} M_j(t)}_{without\ cooperation} - \underbrace{M(K)}_{cooperation} \\ &= \begin{pmatrix} K_1^*(t) & K_2^*(t) & K_3^*(t) \end{pmatrix} \begin{pmatrix} 0 & \epsilon & \delta \\ \epsilon & 0 & \gamma \\ \delta & \gamma & 0 \end{pmatrix}_{Ind(K)} \begin{pmatrix} M_1(t) \\ M_2(t) \\ M_3(t) \end{pmatrix} \end{aligned} \tag{3}$$

with $K_i^*(t) = \varphi_i \tilde{M}_i(t), \tilde{M}_i(t) := [M_i^* - M_i(t)]\,(i = 1,\ldots,n)$ and $K \in Pot(\mathcal{N})$.

In the sequel, we have

$$(B)_{Ind(K)} := A, \text{with} \begin{cases} a_{ij} = b_{ij} & \text{, if } i \in K \text{ and } j \in K \\ a_{ij} = 0 & \text{, otherwise} \end{cases}$$

For the time-dependent grand coalition we get:

$$\begin{aligned} v_t(\mathcal{N}) &:= \underbrace{\sum_{j\in \mathcal{N}} M_j(t)}_{without\ cooperation} - \underbrace{M(\mathcal{N})}_{cooperation} \\ &= \begin{pmatrix} K_1^*(t) & K_2^*(t) & K_3^*(t) \end{pmatrix} \begin{pmatrix} 0 & \omega & \omega \\ \omega & 0 & \omega \\ \omega & \omega & 0 \end{pmatrix} \begin{pmatrix} M_1(t) \\ M_2(t) \\ M_3(t) \end{pmatrix} \end{aligned}$$

For $\tilde{M}_i(t)\varphi_i M_i(t) \geq 0$ $(i, j \in \{1,\ldots,n\})$ the difference between the cooperative and the non-cooperative case is always positive. So we have constructed a reasonable cost-game. Now, the method is that at each time step, this amount is put into a central fund, which can also be used as a feasible set for our control process. In the following we analyse a special allocation principle.

4 The Allocation of the fund and the Kyoto Game

In order to get an intuition of the problem, let us begin with a very simple case where we have only two players. The fund exists and the two players have two alternatives to invest. The coordinate axis is the starting point of the two players. Each actor tries to reach the black square which stands for the level of reductions of emissions mentioned in Kyoto Protocol, Kyoto (1997). For that reason we have a limited time-horizon.

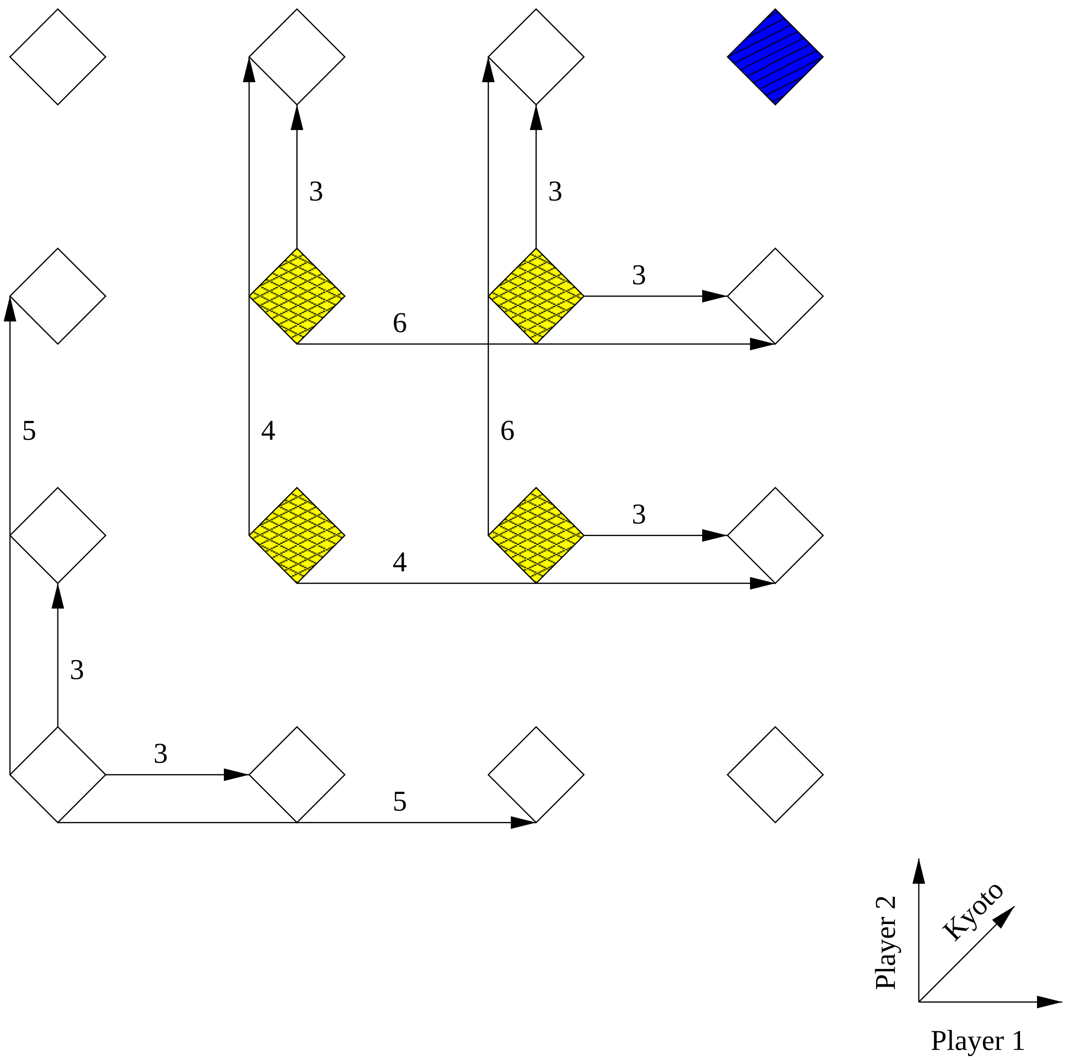

Figure 1: Multi-step investments

After the first time-step one of the squares with a grey square will be attained. The players make their choice independently and simultaneously. The directions are attached by the small diagram. The first player goes to the right. If he reduces one unit of CO_2-emissions, then he has to invest 3 financial units. If he will attain a reduction of two units he has to invest 5 financial units.
One of the main problems in the area of JI and JET is the question of an optimal schedule of technical innovations. Therefore, in Krabs, Pickl (2000) the control problem is solved from a pure mathematical point of view.
Here, we model such a situation with a time-discrete approach where we have three investment units. Each actor can choose between two alternatives (2-step-1-step or 1-step-2-step). The strategy 2-step-1-step stands for a great reduction at the beginning and a smaller investment at the end of the period. The costs are lower than a 1-step-2-step strategy because we want to simulate innovations.
We can transfer this simple model with two players and two time-steps to an easy matrix game, which we call the *Kyoto-Game*.

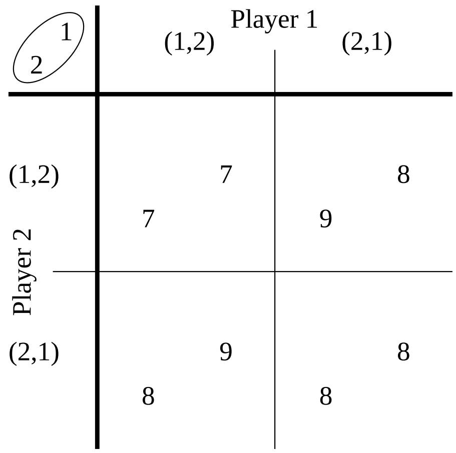

Figure 2: Kyoto Game

The pairs $(7, 7)$ and $(8, 8)$ are Nash-equilibria. A derivation from that chosen strategy is not favourized by one player acting alone. Nevertheless the tuple $((1, 2), (1, 2))$ is favourized by every actor yielding a minimum of financial costs.

5 The Co-funding Process and Joint International Emissions Trading - JET

If we want to support a special path or technology, then we can introduce an additive tax to a special energy path. It is possible to regard ϵ as a *Pigou tax*.

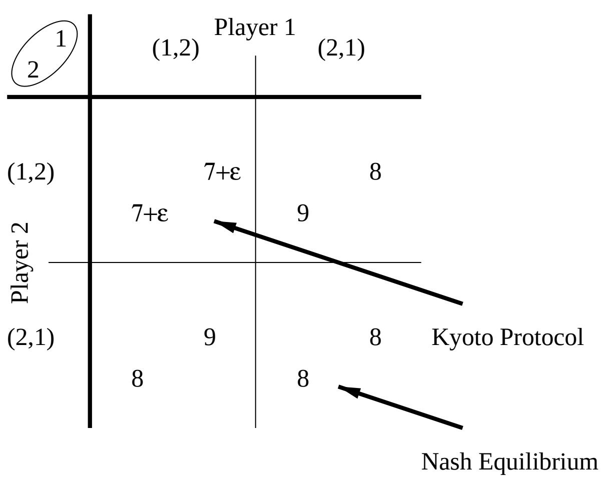

Figure 3: Pigou taxation

It is easy to see that for $\epsilon > 1$ only one Nash-Equilibrium exists. The value $\epsilon = 1$ can be seen as maximal *trading interval*, or as a necessary technical effectivity measure to guarantee uniqueness. Furthermore, as the necessary data is given to the *Clearing House*, we are able to compare the obtained results with real world phenomena. An extension to an n-player situation with various time-steps may lead to new insights in that economic field and support an improvement of such important energy management tools.

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